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Research Article

FL Lyr: Light curve solutions and orbital period analysis by using tess data

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ABSTRACT

The detached binary system FL Lyr, has been controversial based on previous studies; some studies have approved the existence of a third body, and a study with spectroscopic data could not confirm it. Based on new TESS observations, previous results have been recalculated and discussed. We extracted the minimum times from the TESS data and presented a new ephemeris based on the space-based observations by using the MCMC method. Based on the data from Kepler and TESS, the residual O-C diagram shows a sinusoidal period. We estimated the orbital period and the minimum mass of this probable third body to be 10.8 years and 5.6 MJ, respectively. However, due to some reasons mentioned in this study, we cannot confirm the existence of a third body. More photometric and spectroscopic data will be required in the future to analyze the orbital period variation of the FL Lyr binary system for the existence or absence of a third body. Light curve solutions of FL Lyr were done by the PHOEBE python code and the MCMC method. This system did not need to add a star-spot in the light curve solutions, and the system's absolute parameters were also obtained.

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INTRODUCTION

The FL Lyr system is classified as a detached eclipsing binary [1]. In this type of binary, the components are in their own Roche lobe. The stars do not have a significant impact on each other and evolve independently. The orbital period of FL Lyr is 2.1782 days and the magnitude of this system is V_{max}=9.27 which was obtained from the eclipsing variables catalog [2]. The spectral type of the FL Lyr was reported as G0V and F8V in the Malkov et al. [2] and Frasca et al. [3] studies, respectively.

The FL Lyr binary system was discovered in 1935 using photographic plates [4]. The first photometric parameters of the system were determined by Cristaldi [5] and the study also stated some signs of the existence of the third body. Popper et al. [1] studied the radial velocity (RV) curves of this system. The parameters of the components were significantly different from the solution determined by Cristaldi [5]. It was also determined that the sum of masses in the FL Lyr system is ≈ 2M⊙.

The Kepler space telescope monitored the system from 2009 to 2014, with the main goal of finding exoplanets. Kozyreva et al. [6] studied the light-time effect (LITE) in the FL Lyr system. The eclipse timing measurements suggested that there is a third body in the system with an orbital period of ≥ 7 years. The third body has a much longer orbital period than the central binary. The lower limit for the mass of

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the third body was determined, which is around four Jupiter masses. Some of the obtained parameters of the system, especially luminosities, were quite different from those achieved by Popper et al. [1]. Furthermore, Hełminiak et al. [7] could not confirm that there is a third body in a long-period orbit based on their HIgh-Dispersion Echelle Spectrograph (HIDES) data. The mass of the system was improved, and the other parameters were better matched to the study of the system by Kozyreva et al. [6]. The radius accuracy improved slightly over Popper et al. [1], reaching 1.8-2.6 percent. According to Eclipse Timing Variations (ETVs) from the Hełminiak et al. [7] investigation, Hełminiak et al. [8] studied the theory of a third body in a 103-day orbit, and they realized that the observed ETVs were probably because of the evolution of star-spots. In comparison to the previous investigation by Popper et al. [1], the updated parameters such as effective temperatures, R_2, and [M/H], mass and radius agreed better with those determined by Popper et al. [1].

In this study, we reviewed previous studies of the FL Lyr system. Recent TESS data has also been used for this study. We extracted all the TESS minima times and added them to the previous list from the Kepler observations. The light curve analysis was performed according to the latest PHOEBE version and the MCMC approach. The system's orbital period variation was then analysed.

DATASET AND TIMES OF MINIMA

The Transiting Exoplanet Survey Satellite (TESS) surveyed a number of targets, and the data from those observations is now available. TESS obtained new photometric observations data in four sectors: 14, 15, 40, and 41, each with an exposure time of 120 seconds. We used the TESS data to extract minima times and performed a light curve analysis of the FL Lyr binary system. TESS-style curves were extracted from the Mikulski Space Telescope Archive (MAST) using the LightKurve code[1].

Finding the times of minima in the light curves is required before studying orbital period variations. We used a Gaussian distribution model to fit selected portions of the light curves that included the minima to extract the times of minima from the TESS data. We employed MCMC sampling methods to determine the uncertainty of those timings [9, 10]. The code is implemented in Python using the PyMC3 package [11]. We extracted 47 primary and 46 secondary times of minima in the Barycentric Julian Date in the Barycentric Dynamical Time (BJD_{TDB}) from TESS data (Table 1).

We agree with the Kozyreva et al. [6] study that the minimum timings reported from ground-based observations are not accurate enough. Some ground-based observations are synchronous with Kepler observations, but in the O-C diagram they are quite far from Kepler's results. Therefore, we just used space-based observations from TESS and Kepler in our investigation. The Kozyreva et al. [6] study had previously extracted the primary minimum times for Kepler data, and we used the same results after checking their accuracy[2].

LIGHT CURVE SOLUTIONS

The light curve analysis in this study was performed using version 2.3.59 of the PHysics Of Eclipsing BinariEs (PHOEBE[3]). PHOEBE is a powerful code for analyzing light curves of photometric and spectroscopic data of binary systems [12] and it is used in conjunction with the emcee package [13]. In general, for the use of PHOEBE, we should first do a q-search and set the fixed parameters, and then, after the initial analysis is almost optimal, star-spots should be added if necessary. The mass ratio values used in most of photometric or spectroscopic studies of the FL Lyr system are close to each other and we used q and $a\sin(i)$ ($R\odot$) from Hełminiak et al. [8] study for initial parameters. Next, we improved the results using the code of optimization section, and obtained the final results and their uncertainties using the MCMC method.

We first estimated the effective temperature in order to determine the photometric elements of the binary system. The temperature of Gaia DR2 was chosen as a fixed parameter for the primary star and we ran the initial light curve analysis. We then estimated the temperatures and performed the final photometric light curve solutions using the equations from the Kjurkchieva et al. [14] study.

$$T_1 = T_m + \frac{c\Delta T}{c+1} \qquad (1)$$

$$T_2 = T_1 - \Delta T \qquad (2)$$

In equations 1 and 2, T_m is the inlet temperature from Gaia DR2, ΔT temperature difference between two stars, and c is l_2 divided by l_1 from the first light curve analysis. The temperatures obtained from previous photometric and spectroscopic investigations for this system are in good agreement with our results (see Table A1).

Due to Lucy [15] and Rucinski [16], the gravity-darkening and bolometric albedo coefficients of the two components are both set to g_1=g_2=0.32 and A_1=A_2=0.5, respectively. The limb-darkening coefficients were derived as free parameters in the PHOEBE analysis, and the Castelli and Kurucz [17] atmospheric models were used for modeling.

The MCMC method was used to improve the initial analyses and determine the uncertainty values. We employed 46 walkers and 1200 iterations for each walker in the MCMC method. We adjusted the $R_{equiv(1)}$, $R_{equiv(2)}$, $a\sin(i)$, q, T_1, T_2, P, and inclination (i) parameters for the MCMC process.

Accordingly, we then have the new orbital period in this study, and we can calculate the total mass of the system by the relation of Kepler's third law. Then, by using the mass ratio, the mass of each component can be estimated. Ac-

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[1] https://docs.lightkurve.org

[2] We used 0.00001 for the uncertainty of Kepler's primary minima because the uncertainty was not mentioned in the Kozyreva et al. [6] study.

[3] http://phoebe-project.org

Table 1. The primary and secondary minima times of FL Lyr from TESS data.

Min. (BJD_{TDB})	Error	Epoch	O-C	Min. (BJD_{TDB})	Error	Epoch	O-C
2458685.31174	0.00006	1713	0.00123	2458684.22244	0.00007	1712.5	0.00100
2458687.48987	0.00003	1714	0.00120	2458686.40060	0.00005	1713.5	0.00101
2458689.66802	0.00004	1715	0.00120	2458688.57874	0.00005	1714.5	0.00100
2458691.84611	0.00004	1716	0.00113	2458690.75672	0.00004	1715.5	0.00082
2458694.02428	0.00003	1717	0.00115	2458692.93497	0.00004	1716.5	0.00092
2458696.20234	0.00004	1718	0.00105	2458695.11322	0.00005	1717.5	0.00101
2458698.38061	0.00003	1719	0.00117	2458699.46967	0.00004	1719.5	0.00115
2458700.55878	0.00004	1720	0.00119	2458701.64782	0.00005	1720.5	0.00115
2458702.73693	0.00004	1721	0.00118	2458703.82600	0.00005	1721.5	0.00118
2458704.91500	0.00004	1722	0.00110	2458706.00414	0.00006	1722.5	0.00116
2458707.09310	0.00004	1723	0.00105	2458708.18218	0.00005	1723.5	0.00105
2458709.27126	0.00004	1724	0.00105	2458712.53847	0.00004	1725.5	0.00103
2458713.62766	0.00004	1726	0.00115	2458714.71674	0.00006	1726.5	0.00114
2458715.80585	0.00004	1727	0.00118	2458716.89492	0.00005	1727.5	0.00117
2458717.98402	0.00003	1728	0.00120	2458719.07307	0.00005	1728.5	0.00117
2458720.16215	0.00005	1729	0.00117	2458721.25134	0.00005	1729.5	0.00128
2458722.34031	0.00004	1730	0.00118	2458723.42947	0.00005	1730.5	0.00126
2458726.69658	0.00004	1732	0.00114	2458725.60761	0.00006	1731.5	0.00125
2458728.87474	0.00003	1733	0.00114	2458727.78576	0.00006	1732.5	0.00124
2458731.05295	0.00004	1734	0.00121	2458729.96400	0.00006	1733.5	0.00133
2458733.23109	0.00004	1735	0.00119	2458732.14214	0.00006	1734.5	0.00132
2458735.40926	0.00003	1736	0.00121	2458734.32031	0.00006	1735.5	0.00133
2459391.03387	0.00003	2037	0.00146	2458736.49848	0.00006	1736.5	0.00135
2459393.21192	0.00003	2038	0.00135	2459392.12273	0.00004	2037.5	0.00124
2459395.39003	0.00003	2039	0.00131	2459394.30089	0.00005	2038.5	0.00125
2459397.56826	0.00003	2040	0.00139	2459396.47904	0.00005	2039.5	0.00125
2459399.74639	0.00004	2041	0.00136	2459398.65725	0.00005	2040.5	0.00130
2459401.92465	0.00003	2042	0.00147	2459400.83537	0.00005	2041.5	0.00127
2459404.10271	0.00003	2043	0.00138	2459403.01354	0.00005	2042.5	0.00128
2459406.28093	0.00003	2044	0.00145	2459407.36972	0.00005	2044.5	0.00115
2459408.45907	0.00003	2045	0.00142	2459409.54790	0.00005	2045.5	0.00118
2459410.63719	0.00003	2046	0.00140	2459411.72597	0.00005	2046.5	0.00110
2459412.81535	0.00003	2047	0.00140	2459413.90423	0.00005	2047.5	0.00120
2459414.99347	0.00003	2048	0.00137	2459416.08200	0.00005	2048.5	0.00081
2459417.17165	0.00003	2049	0.00140	2459418.26054	0.00005	2049.5	0.00120
2459421.52782	0.00003	2051	0.00125	2459420.43885	0.00004	2050.5	0.00136
2459423.70594	0.00004	2052	0.00122	2459422.61690	0.00004	2051.5	0.00126
2459425.88408	0.00004	2053	0.00121	2459424.79510	0.00005	2052.5	0.00130
2459428.06226	0.00004	2054	0.00123	2459426.97332	0.00005	2053.5	0.00137
2459430.24042	0.00003	2055	0.00124	2459429.15152	0.00005	2054.5	0.00142
2459432.41809	0.00005	2056	0.00076	2459431.32977	0.00005	2055.5	0.00151
2459434.59681	0.00003	2057	0.00132	2459435.68627	0.00005	2057.5	0.00170
2459436.77500	0.00003	2058	0.00135	2459437.86447	0.00006	2058.5	0.00175
2459438.95315	0.00003	2059	0.00135	2459440.04265	0.00006	2059.5	0.00177
2459441.13133	0.00003	2060	0.00138	2459442.22076	0.00005	2060.5	0.00173
2459443.30952	0.00004	2061	0.00142	2459444.39898	0.00005	2061.5	0.00180
2459445.48766	0.00004	2062	0.00140				

cording to asin(i) (R⊙) and inclination, a can be calculated, and then the radius of each star can be obtained based on equation 3. Having the temperature and radius of each star, their luminosity was calculated from equation 4. Eventually, we could calculate Mbol of stars by using Pogson's [18] relation (Equation 5), and the surface gravity was determined by the well-known relation (Equation 6).

$$R = a \times r \tag{3}$$

$$L = 4\pi R^2 \sigma T^4 \tag{4}$$

$$M_{bol} - M_{bol\odot} = -2.5\log\left(\frac{L}{L_\odot}\right) \tag{5}$$

$$g = G_\odot\left(\frac{M}{R^2}\right) \tag{6}$$

Table A1. The FL Lyr's light curve solutions and absolute parameters from previous studies.

Parameter	Cristaldi (1965)	Giannone & Giannuzzi (1974)	Botsula (1978)	Cester et al. (1979)	Popper et al. (1986)	Anderson (1991)	Svechnikov & Perevozkina (1999)	Guillout et al. (2009)
T_1(K)		5984			6150(100)	6152(100)	6026(2102)	
T_2(K)		4932			5300(100)	5296(98)	5495(1918)	
q	1.169				0.786			
i(deg)	89.0		87.1(3)		86.3(4)			
l_2/l_1					0.249(16)			
asin(i) ($R_\odot$)					9.15(4)			
r_1(mean)	0.132		0.102(3)		0.140(3)			
r_2(mean)	0.114		0.134(4)		0.105(3)			
M_1($M_\odot$)	1.0	1.02	1.26	0.98(13)	1.218(16)	1.221(16)	1.22(2)	1.218(16)
M_2($M_\odot$)	1.1	0.93	0.85	0.03(1)	0.958(11)	0.960(12)	0.96(2)	0.958(12)
R_1($R_\odot$)	1.19		1.39	0.81(10)	1.283(30)	1.282(28)	1.28(2)	
R_2($R_\odot$)	1.03		1.03	0.94(9)	0.963(30)	0.962(28)	0.96(2)	
L_1($L_\odot$)	0.234	1.15	0.791(11)		2.168(182)	2.089(150)	2.042(713)	
L_2($L_\odot$)	0.430	0.71	0.209(11)		0.653(66)	0.661(64)	0.759(265)	
log g_1(cgs)					4.31(2)	4.309(20)	4.31(5)	4.307(21)
log g_2(cgs)					4.45(3)	4.454(26)	4.46(6)	4.453(28)
P (day)	2.1781544	2.178			2.1781542(3)			
T_1(K)		6152(100)	6150(100)		6252(116)		6500(150)	6260(120)
T_2(K)		5233(18)	5300(95)		5495(246)		5600(100)	5490(240)
q		0.78(1)					0.7888(22)	0.7860(23)
i(deg)		85.6(2)		85.9			85.36(71)	87.13(71)
l_2/l_1						0.2646	0.224(35)	0.224(35)
asin(i) ($R_\odot$)							9.105(14)	9.134(15)
r_1(mean)				0.123			0.1389(25)	0.1361(25)
r_2(mean)				0.123			0.0995(27)	0.0984(26)
M_1($M_\odot$)	1.173	1.23(1)	1.218(16)		1.210(8)		1.2041(76)	1.2102(76)
M_2($M_\odot$)	0.815	0.96(1)	0.958(11)		0.951(4)		0.9498(46)	0.9512(39)
R_1($R_\odot$)	1.267	1.20(2)	1.282(30)		1.244(23)	1.283(30)	1.269(23)	1.244(23)
R_2($R_\odot$)	0.913	1.05(1)	0.962(30)		0.900(24)	0.962(30)	0.908(24)	0.900(24)
L_1($L_\odot$)		2.17(13)	2.109(182)	0.596	2.138(523)			2.138(179)
L_2($L_\odot$)		0.65(4)	0.656(68)	0.298	0.661(134)			0.661(134)
log g_1(cgs)			4.308(22)		4.331(16)		4.312(16)	4.331(16)
log g_2(cgs)			4.453(28)		4.508(23)		4.499(23)	4.508(23)
P (day)							2.17815425(7)	2.17815425(7)

The results of the light curve solutions and absolute parameters in this investigation are shown in Table 2, while the results of previous literature are shown in the appendix (Table A1). Figure 1 shows the observed and final synthetic light curves for the FL Lyr system. There was no need to add a star-spot while analyzing the light curves. The geometrical structure of the stars in this system is shown in Figure 2.

ORBITAL PERIOD VARIATIONS

The FL Lyr binary system's period variations were investigated using the timings of minima acquired from Kepler and TESS space-based observations. The minimums obtained from ground-based observations are highly scattered compared to space-based observations. Therefore,

Table 2. The results of FL Lyr's light curve solutions and absolute parameters in this study

Parameter	Result	Parameter	Result
T_1(K)	6107 (-19) (+28)	M_1($M_\odot$)	1.213 (-2) (+1)
T_2(K)	5191 (-15) (+24)	M_2($M_\odot$)	0.955 (-10) (+7)
q	0.787 (-8) (+6)	R_1($R_\odot$)	1.276 (-15) (+15)
i(deg)	86.48 (-5) (+16)	R_2($R_\odot$)	0.932 (-9) (+12)
l_1/ltot	0.777(1)	L_1($L_\odot$)	2.032 (-72) (+86)
l_2/ltot	0.223(1)	L_2($L_\odot$)	0.566 (-17) (+25)
asin(i) ($R_\odot$)	9.133(-13) (+12)	log g_1(cgs)	4.310 (-10) (+10)
r(mean)$_1$	0.139 (1)	log g_2(cgs)	4.479 (-4)(+8)
r(mean)$_2$	0.102 (1)	a($R_\odot$)	9.150 (-12)(+11)

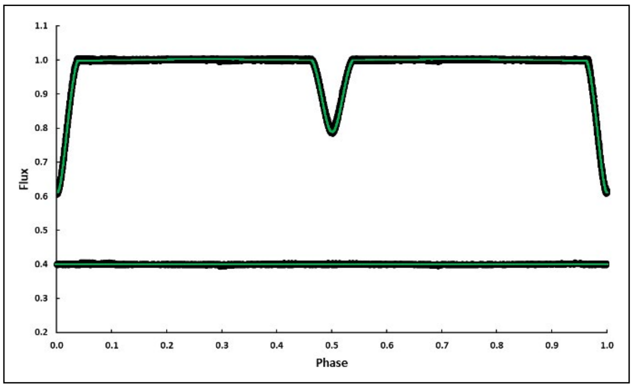

Figure 1. Observational light curve from TESS data (black dots) and the synthetic light curve obtained from the model (green line).

we ignored all ground-based observations in this investigation. The O-C diagram shows the difference between the observed minimum times (T_0) and the predicted minimum times (T_c), which are calculated from the following relation:

$$\delta_t = T_0 - T_c \quad (7)$$

Changes in the O-C diagram are usually shown by using relation 8:

$$\delta_t = (\Delta T_0 + \Delta PE) + QE^2 + \delta T_i \quad (8)$$

The phrase in parenthesis in relation 8 indicates the linear variation of data in the O-C diagram caused by uncertainties in obtaining the orbital period and reference minimum time. Also, QE^2 is related to mass transfer in the contact binary systems, δT_i due to physical phenomena such as light time travel due to the presence of a third body, apsidal motion, magnetic activity of the binary system, etc., which cause more complex periodic changes in determining the orbital period time of the system [19]. We provided a linear fit to the data (Table 1) using the MCMC method (100 walkers, 10000 step number, and 200 burn-in) using the emcee package in Python [13] to refining the linear change in the system's orbital period. Figure 3 shows the linear fit diagram. As a result, the reference ephemeris from the Kepler Eclipsing Binary Catalog (KEBC[4]) [8] was modified, and a refined ephemeris was calculated.

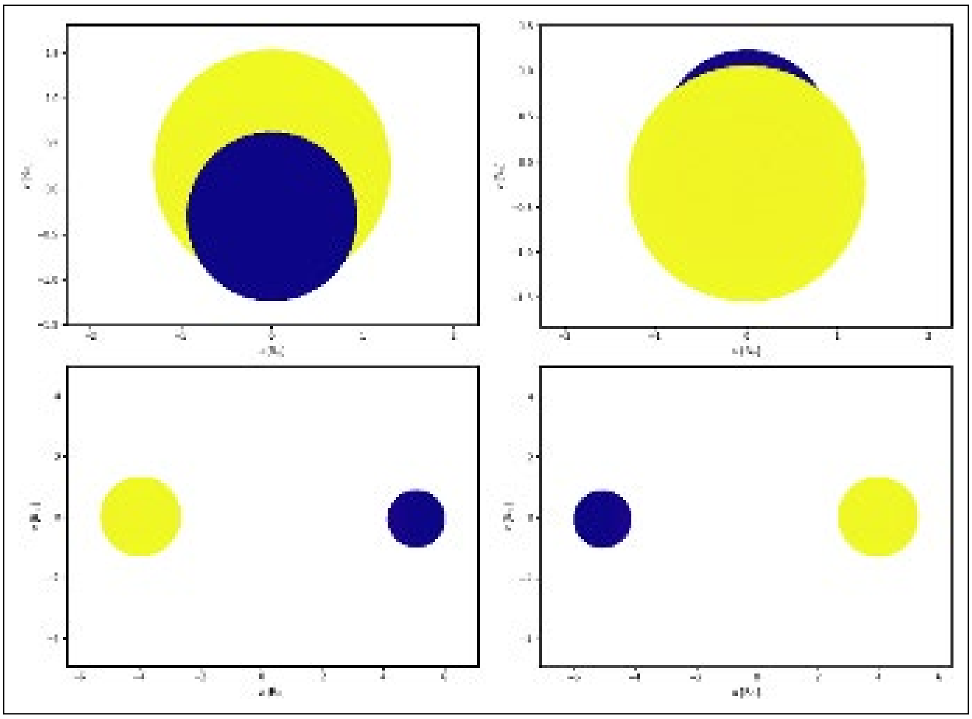

Figure 2. The 3D view of the stars in different phases.

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[4] http://keplerebs.villanova.edu

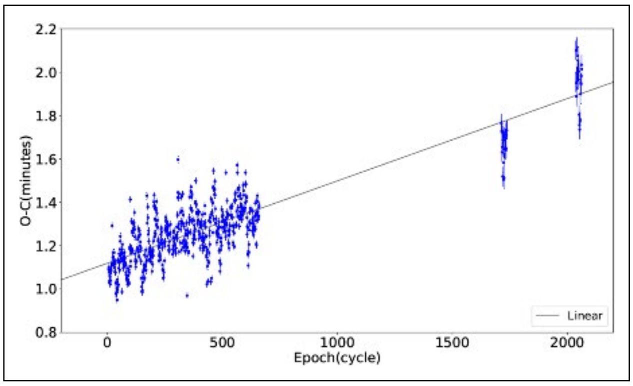

Figure 3. The initial O-C diagram of FL Lyr with a linear trend in the data derived by the MCMC approach.

$$Reference\ ephemeris\ (BJD_{TDB}) = (2454954.132713 \pm 0.0002) + (2.178154 \pm 0.000003) \times E \quad (9)$$

$$New\ Ephemeris\ (BJD_{TDB}) = (2454954.1334898 \pm 0.0000025) + (2.17815426 \pm 0.00000008) \times E \quad (10)$$

Then, by subtracting the linear slope obtained in the O-C diagram, we draw the residual O-C diagram. This diagram shows regular periodic changes over several years. The apsidal motion phenomena cannot account for these sinusoidal changes since the FL Lyr binary system's eccentricity is nearly zero. Also, studies of the magnetic activity of this system show periodic changes in the O-C diagram at intervals of about 100 days, which is related to the effect of star-spots, which due to the short period, cannot justify long-term sinusoidal changes over several years. Kozyreva et al. [6] used observations from the Kepler mission to investigate the presence of a third body in the FL Lyr system. The results of Kozyreva et al. [6] appeared to show the existence of a third body with an orbital period of 7.2 years in the FL Lyr system. Furthermore, Hełminiak et al. [7] and Hełminiak et al. [8] using the data obtained from the results of the radial velocity of the FL Lyr system investigated the existence of a third body in the system. The orbital period extracted from the O-C diagram in their study is 103.2 days, which is due to the presence of star-spots in the binary system, and was also studied by Yoldaş and Dal [20]. It should be noted that investigations into the radial velocity curve did not confirm the existence of a third body.

We reanalyzed the LITE phenomenon adding TESS observations. We have completed the O-C diagram with the TESS space telescope observations.

The final fit of our residual O-C diagram displays a sinusoidal period variation, just like the Kozyreva et al. [6] study. If a third body were to be the source of these changes, its orbital period would be 10.8 years and its maximum diagram change would be δT_{max}=7.5 seconds. According to Figure 4, it is assumed the argument of periapsis (ω) and the eccentricity of the third mass circuit around the binary system is close to zero. Also, the value of asin(i) related to the motion circuit of the binary system is obtained from relation 11 as equal to 0.0156 AU, where c is the speed of light.

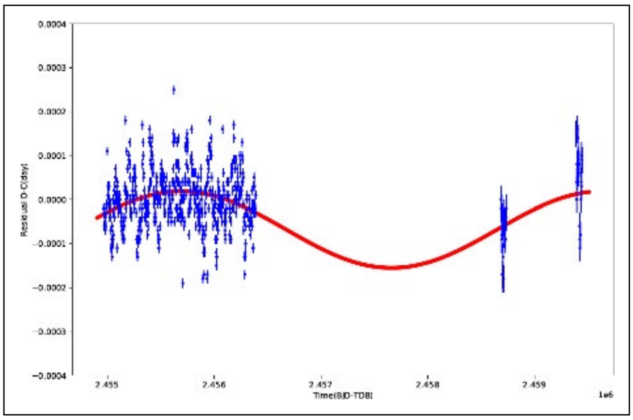

Figure 4. The Residual O-C diagram of FL Lyr based on the TESS and Kepler observations.

$$\delta T_{max} = \frac{a sini\sqrt{1-e^2 cos^2\omega}}{c} \quad (11)$$

The mass function of the system is then calculated at 3.27×10^{-8} M⊙ using relation 12. So, the minimum mass of the third body if exists is 5.6 M_J.

$$f(m_3) = \frac{m_3^3 sin^3 i}{(m_1+m_2+m_3)^2} \quad (12)$$

SUMMERY AND CONCLUSION

Light curve solutions of FL Lyr was achieved by using PHOEBE with the MCMC method on the photometric TESS data. Given the estimated presence of a third body in this system in previous studies and its long orbital period, we do not expect the third body to be revealed in light curve analysis. The PHOEBE method with the MCMC method seems to be powerful, and the results are close to the results of the previous spectroscopic analysis. In all the light curves observed in four sectors of the TESS data, there was no need to add any star-spots in the light curve analysis. The temperature difference between the two stars is in the range of 1000 K. According to the temperature, the components' spectral types can be estimated based on the Eker et al. [21] study, which is F8 for the primary star and K0 for the secondary star.

According to the estimated luminosity, it is possible to obtain the absolute magnitude of each component. Therefore, with A_v=0.04 [22], and V=9.67 [23], we measured the distance as 135.5±2.5 pc. The Gaia DR3 parallax gives a distance value of 134.1±0.2 pc, thus, our estimated distance for this binary system appears to be compatible with the Gaia DR3 distance.

The diagrams of Mass-Luminosity (M-L) and Mass-Radius (M-R) show the theoretical zero-age main sequence (ZAMS), terminal-age main sequence (TAMS) lines, and the positions of the primary and secondary components. Figure 5 shows that both stars are in the main sequence.

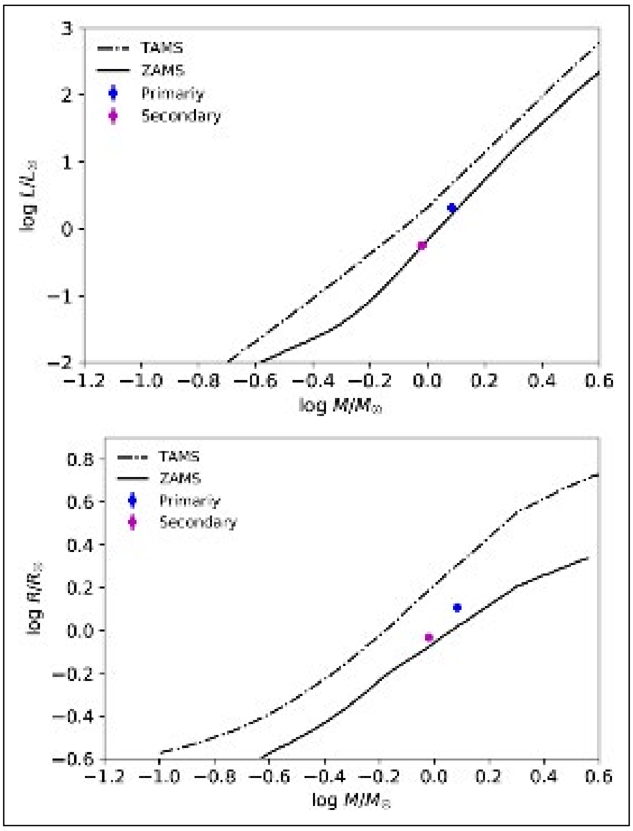

Figure 5. The M-L and M-R diagrams for FL Lyr. The TAMS and ZAMS are shown, and the positions of the FL Lyr components are indicated.

In previous studies, Kepler data has been used. By combining the results of the data from Kepler and TESS space telescopes, we were able to analyze the system's periodic orbital changes. The orbital period variations caused by the LITE phenomenon were investigated because of the suggested existence of a third body in the system.

We extracted times of minima from TESS data by a Gaussian distribution model and employed the MCMC to determine the value's uncertainty. We used 47 primary times of minima for this study and added them to Kepler times of minima from a previous study. Therefore, we presented a new ephemeris by using the MCMC sampling.

A periodic changes over several years were shown in the residual O-C diagram. If there is a third body in this binary system, we calculated its periodicity, which is longer than in the Kozyreva et al. [6] study. We have also obtained its estimated mass, which shows that it could be a substellar object.

For the following reasons, we are unable to confirm the existence of a third body in this binary system for now:

A) We were limited to use only space-based data in this work since there are no reliable ground-based observations for the FL Lyr system. Hence, there is a big gap between the Kepler and TESS observations since there are no data for the interval between about epoch 700 and 1700 cycles in the O-C diagram. There is cause to believe that it may be a linear fitting if there are enough data to fill the interval.

B) We plotted a sinusoidal fitting in the Residual O-C diagram in Figure 4. However, the amplitude of the sinusoidal variation is smaller than the error bars of the O-C data.

Eventually, identifying the cause of the observed eclipse timing variation is highly dependent on future studies with more accurate spectroscopic instruments.

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AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work

DATA AVAILABILITY STATEMENT

The authors confirm that data that supports the findings of this study are available within the article.

The graphs or raw data that support the findings of this study can be requested from the corresponding author.

CONFLICT OF INTEREST

The authors declared no potential conflicts of interest with regards to research, authorship, and/or publication of the article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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